◇論文◇

# Variable Wage and the Switching of Techniques:
## A Wicksellian Approach to Ricardo's Machinery Chapter*

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Following Wicksell (1934), this paper reformulates Ricardo's (1951, ch.31) argument on the effects of the mechanization on workers. As a result, it is shown that, in the short run, the advanced wage can continue to be below its initial value when the new technology with the machine is introduced, while, in the long run, it can continue to exceed the initial value. Thus Ricardo's argument can hold: An introduction of new technology with the machinery is injurious to workers in the short run, but beneficial in the long run.

## 1 Introduction

In Chapter 31 of the third edition of his *Principles*, "On Machinery," using a numerical example, David Ricardo showed that the switching from the technique using the labour alone to one using both the labour and the machine can decrease the demand for labour or the wage fund (Ricardo, 1951, pp.388-9). However, he did not fully explain the subsequent effects of the decrease in the demand for labour on *the level of wage and the choice of techniques*.

With respect to the effects, Knut Wicksell pointed out as follows:

> For as soon as a number of labourers have been made superfluous by these changes, and wages have accordingly fallen, then, as Ricardo failed to see, the old methods of production ...... will become more profitable; they will develop, using labour more intensively and absorb the surplus of idle labourers. (Wicksell, 1934, p.137).

As Wicksell says, if the switching of techniques depends on the wage, and if the wage depends on the demand and supply for labour, then, even in the next period of the switching (if we adopt the discrete-time model), the reswitching of techniques may come about under Ricardo's wage-fund setting: suppose that the higher wage in period $t$ causes the switching of techniques toward one using the machine more intensively, that the switching in period $t$ causes a decrease

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of the output of the food in period $t$, and that the output composes the real wage fund in period $t+1$. Then the wage fund in period $t+1$ (=the output of the food in period $t$), namely the demand for labour in period $t+1$, is smaller, and the supply for labour in period $t+1$ is larger (according to the Malthusian population adjustment mechanism and the higher wage in period $t$), so that the wage in period $t+1$ is lower, and the technique using the labour more intensively will be chosen again in period $t+1$.

Following Wicksell, this paper examines the interactions between the changes of wage under Ricardo's wage-fund setting and the switching (or reswitching) of techniques, which Ricardo did not fully explain in the machinery chapter. Adopting this approach, we can value, from the viewpoint of the level of wage, Ricardo's argument that the introduction of new technology with the machinery is injurious to the labourers in the short run, but beneficial in the long run (Ricardo, 1951, p.390). Hollander (1987, p.189) also points out the possibility of this type of approach.

In this paper formalizing the switching of techniques or the substitution between factors in Ricardo's machinery chapter, the cost-minimization (or profit-maximization) behavior is assumed. This is based on the following Ricardo's words in the machinery chapter:

> [1] If I employed one hundred men on my farm, and if I found that the food bestowed on fifty of those men, could be diverted to the support of horses, and afford me a greater return of raw produce, after allowing for the interest of the capital which the purchase of the horses would absorb, it would be advantageous to me to substitute the horses for the men, and I should accordingly do so; (Ricardo, 1951, p.394).

> [2] Machinery and labour are in constant competition, and the former can frequently not be employed until labour rises. (Ricardo, 1951, p.395).

These will imply that Ricardo considered the switching of techniques or the substitution between factors from the viewpoint of the cost-minimization (or profit-maximization).

On the other hand, Ricardo seems to assume the fixed-wage in Chapter 31. Most of Ricardian models to formalize Ricardo's argument on machinery also assume the fixed-wage. For instance, Hicks (1969, Appendix), Brems (1970), Barkai (1986), Samuelson (1989), Shields (1989), Negishi (1990), and Uchiyama (2000).[1] However, Ricardo will never deny the possibility of the above Wicksellian scenario:

> The market price of labour is the price which is really paid for it, from the natural operation of the proportion of the supply to the demand; labour is dear when it is scarce, and cheap when it is plentiful. (Ricardo, 1951, p.94).

1 Samuelson (1988, 1994) formalizes Ricardian machinery models with variable wage. However, dynamics of interactions between the changes of wage and the switching of techniques is not explicitly examined. As a result, the analyses are almost the same as one by the comparative statics under the fixed-wage.

I also note that in the quotation [2], Ricardo thought, the change of the wage is the cause of the switching of techniques or the substitution between factors, so that the fixed-wage setting may not essentially be suited to the machinery chapter. Thus, following Wicksell, I also assume the non-instantaneous Malthusian population adjustment (NIM) mechanism, under which the wage is variable and determined by the demand and supply for labour. This mechanism is often used in Ricardian models without the machinery. For instance, Hicks and Hollander (1977), Casarosa (1978), Maneschi (1983), and Costa (1985). This paper introduces NIM into Ricardian model with the machinery.

In the above Wicksellian scenario there are some questions to be answered. First is whether the higher (lower) wage always cause the use of the technique using the machine (labour) more intensively, because the higher (lower) wage may cause a rise (fall) of the machine price:

> I hope I have succeeded in showing, ...... that only those commodities would rise which had less fixed capital employed upon them than the medium in which price was estimated, and that all those which had more, would positively fall in price when wages rose. (Ricardo, 1951, p.46).

If the machine has "less fixed capital employed upon them than" numeraire, the price of the machine "would rise" (fall) "when wages rose" (fall), so that the labour may be relatively cheaper (dearer) than the machine, and the technique using the labour (machine) more intensively may be chosen even when the wage rises (falls).

Second is whether the output of the wage good, which composes the wage fund, always decrease under the technique using the machine more intensively, which is chosen for the higher wage.[2] If the output of the wage good increases, the demand for labour (the wage fund) expands, and the wage may not fall.[3]

Third is whether the long run wage is unique and dynamically stable, and hence whether the technique chosen in the long run is also so.

Forth is what intertemporal tendencies the interactions between the mechanization and the wage under the wage-fund setting show. This may help us to judge the validity of the above Ricardo's argument on the effects of mechanization on the workers.

To answer the above questions, I employ a Ricardian two-good (the food and the machine), two-factor (the labour and the machine) model with NIM mechanism. A crucial and common feature of most of Ricardian machinery systems including Hicks (1969, Appendix), Brems (1970), Barkai (1986), Shields (1989), Negishi (1990), Uchiyama (2000), and this paper is the absence of the land, the rent, and the landlords. In fact they are neglected in Ricardo's

2 See Uchiyama (2000) for the argument under the fixed-wage setting.

3 In criticizing Ricardo on machinery, Wicksell (1934, pp.137-41) insisted that the output of the wage good always expands after a fall of wage due to the decrease in the demand for labour. See Samuelson (1989) for the validity of Wicksell's criticism of Ricardo.

machinery chapter. This structure without the fixed input is very similar to one of the neoclassical two-sector growth models such as Uzawa (1961).

Using my Ricardian two sector machinery model, I show the followings. A rise of the advanced wage always causes the use of the technique using the machine more intensively even when the rise of the advanced wage causes a rise of the machine price. Given the labour and the machine stock, a rise of the advanced wage causes a decrease in the output of the food if and only if the food sector is machine intensive. It is necessary for the dynamic stability of the advanced wage and the technique in the long run equilibrium (steady state) that the food sector is machine intensive. There can be the dynamic path to the steady state on which, from the viewpoint of the level of wage, the above Ricardo's argument on the effects of mechanization holds, although there is also the path on which his argument does not hold.

In Section 2, I describe the basic structure and notation of the model. In Section 3, using simple figures, I examine the properties of the momentary equilibrium where the factor endowments are given. In Section 4, I examine the effects of the changes of the advanced wage and the factor endowments on the outputs. In Section 5, I examine the dynamic properties of the system. Section 6 concludes.

## 2 Ricardian machinery system

The system consisting of two classes, capitalists and workers, uses two factors, labour and machine, to produce two goods. Good 1 is food and the system's numeraire, and good 2 is machine. Let $p$ denote the relative price of the good 2, the machine, in terms of the good 1, the food.

Capitalists play two roles: the roles as the managers and the owners of capitals.[4] With the former, capitalists maximize profit for given factor prices, but they have no profit under the perfect competition. With the latter, they earn rental prices of capitals, which are only income of capitalists under the perfect competition. Of course, a capitalist may play both roles. I call capitalists' income under the perfect competition (namely rental prices of capitals) the profit income as in the literature for the classical economics.

To advance market food wage, $\omega$, to workers at the beginning of each production period, capitalists as managers borrow it from capitalists as owners of the real wage fund composed of food, $W$, and return it with its rental price at the end of each production period. However, since the real wage fund borrowed by the capitalists as managers completely disappears as the advanced payment to workers, the capitalists as managers must pay the capitalists as owners of the wage fund the compensation for the full depreciation. Note that the advanced payment to workers is dealt with as a kind of the depreciation of capital (the wage fund). Thus the cost for the capitalists as the managers to employ the workers is the rental price of the wage fund plus the compensation for the depreciation of the wage fund. Similarly for the machine,

4 In Costa (1985), the capitalists as the managers are called the entrepreneurs. As he says, this distinction of the capitalists' roles is useful to clear the analysis.

capitalists as managers borrow it from capitalists as owners of the machine stock, $K$, and return it with its rental prices. To simplify the argument and the notation, I assume that the machine has the infinite durability, so that the compensation for the depreciation of the machine stock is zero. Let $w_L$ and $w_K$ denote the costs for the capitalists as the managers to employ one unit of the labour and the machine respectively. From the above argument, the factor prices of the labour and the machine, $w_L$ and $w_K$, are composed of as follows:

$$w_L = (1+r)\omega \tag{1}$$

$$w_K = rp, \tag{2}$$

where $r$ is the rental price per value of capitals (the wage fund and the machine stock), namely the uniform rate of profit.

Technology is represented by the production functions. $F_i(L_i,K_i)$ is the production functions of the good $i$, where $L_i$ and $K_i$ are respectively the amounts of labour and machine employed in the good $i$ $(i=1,2)$ sector. The neoclassical production functions which allow the smooth substitution between the land and the labour are often used in Ricardian models without the machinery: for instance, Pasinetti (1960), Findlay (1974), Hicks and Hollander (1977), Casarosa (1978), Maneschi (1983), Costa (1985), Burgstaller (1986, 1989), Samuelson (1989, Mathematical Appendix), Negishi (1989, ch.4), and Uchiyama (2005), although, in all Ricardian models except for Burgstaller (1986, 1989), Samuelson (1989, Mathematical Appendix), and Uchiyama (2005), the land as an input does not explicitly appear in the production functions. On the other hand, from the quotations [1] and [2], it is evident that in the machinery chapter Ricardo took into account the possibility of the substitution between the labour and the machine.[5] Thus, to deal with the switching of the techniques or the substitution between the labour and the machine explicitly and easily, following the above Ricardian models without the machinery, I also employ the standard neoclassical production functions which allow the smooth substitution between factors. Namely each production function, $F_i(\mathrm{i}=1,2)$, is increasing, concave, linearly homogeneous, differentiable up to the necessary order in inputs, and satisfies Inada conditions, as in the standard neoclassical models. Let $c_i(w_L, w_K)$ denote the unit cost function of the good $i(i=1,2)$. By Shephard's lemma, the cost-minimizing amount of the factor $j$ to produce one unit of the good $i$ is equal to $c_{ij} \equiv \partial c_i / \partial w_j (i=1, 2, j=L, K)$.[6]

I assume that the factor intensity (namely machine/labour ratio), $c_{iK}/c_{iL} \equiv k_i$, is different across the goods 1 and 2 sectors, $k_1 \neq k_2$ for any $w_j (j=L, K)$, and that there is no factor intensity reversal. If the factor intensities in both sectors are identical, the meanings of two-factor setting are lost. Furthermore by the assumption we can consider technology in Ricardian machinery models such as Hicks (1969, Appendix), Brems (1970), Barkai (1986), Shields (1989),

5 Horses in quotation [1] can be regarded as a kind of machines or robots. See Samuelson (1988). Note that quotation [1] is located in the machinery chapter.

6 For the Ricardian models using the duality approach, see, for instance, Burgstaller (1989) and Uchiyama (2005).

Negishi (1990), and Uchiyama (2000), where the food is produced by the machine and the labour while the machine is produced by the labour alone, as a special case of our model: $k_1 > 0 = k_2$.

The zero excess profit conditions for capitalists as the managers under the perfect competition are

$$1 = w_L c_{1L}(w_L, w_K) + w_K c_{1K}(w_L, w_K), \tag{3}$$

$$p = w_L c_{2L}(w_L, w_K) + w_K c_{2K}(w_L, w_K). \tag{4}$$

On the other hand, the total labour, $L$, the wage fund, $W$, and the machine stock, $K$, are the momentarily predetermined state variables of the system, and $W$ and $K$ compose capital stocks. Thus the full employment conditions of the factors, the labour and the machine stock, are

$$L = c_{1L}(w_L, w_K) Y_1 + c_{2L}(w_L, w_K) Y_2, \tag{5}$$

$$K = c_{1K}(w_L, w_K) Y_1 + c_{2K}(w_L, w_K) Y_2, \tag{6}$$

where $Y_i$ is total output of the good $i(i=1, 2)$. The advanced market food wage, $\omega$, is determined by the wage fund theory:

$$L = W/\omega. \tag{7}$$

Note that, under the instantaneous Malthusian population adjustment mechanism in Pasinetti's (1960), Findlay's (1974), Burgstaller's (1986, 1989), and Uchiyama's (2005) Ricardian models, $L$ is instantaneously adjusted to satisfy (7) for given $W$ and constant $\omega$. Under our NIM mechanism as in Hicks and Hollander (1977), Casarosa (1978), Maneschi (1983), and Costa (1985), the market food wage, $\omega$, is adjusted to satisfy (7), namely the equality of the labour demand ($W/\omega$) and the labour supply ($L$), for given $W$ and $L$.

Finally, to endogenize the changes of system's state variables, $L$, $W$, and $K$, we must specify the workers' and the capitalists' behaviors. I assume that the workers do not save at all, and that the capitalists invest all of their profit income, $r(W+pK)$ in the capitals (the wage fund and the machine stock), following Pasinetti (1960), Findlay (1974), and Negishi (1989, ch.4). This assumption is also Uzawa's (1961). It is called the classical saving behavior. On the other hand, the labour expands (contracts) if the market food wage is higher (lower) than the subsistence or natural food wage level, $\bar{\omega}$, which is a positive constant given exogenously. The labour does not change if the market food wage is equal to the subsistence level. Further, for given market food wage, an exogenous rise of the subsistence level causes a decrease in the labour. The above is called the Malthusian population adjustment mechanism. Thus system's state variables, $L$, $K$, and $W$ change according to the following dynamic equations: (Let $\dot{m}$ denote $dm/dt$ of any variable $m$, where $t$ is time.)

$$\dot{L}/L = g(\omega, \bar{\omega}), \tag{8}$$

$$\dot{K} = Y_2, \tag{9}$$

$$\dot{W} + p\dot{K} = r(W + pK), \tag{10}$$

where $g_1 \equiv \partial g/\partial \omega > 0 = g(\bar{\omega}, \bar{\omega}) > g_2 \equiv \partial g/\partial \bar{\omega}$, $g(\cdot)$ is the change rate per time of the labour, and $\dot{W}$ and $\dot{K}$ are the net investments of the wage fund and the machine stock respectively, and (10) is the capitalists' budget constraint. From (1)-(7), (9), and (10), we have

$$\dot{W} = Y_1 - W. \tag{11}$$

We have ten equations, (1)-(10), and ten unknowns, $w_L$, $w_K$, $r$, $p$, $\omega$, $Y_1$, $Y_2$, $\dot{L}$, $\dot{W}$, and $\dot{K}$, under system's state variables, $L$, $W$, and $K$. Thus we can characterize fully the intertemporal behavior of the above Ricardian system if the initial values of $L$, $W$, and $K$ are given.

A crucial feature of the above Ricardian machinery system is that, given factor endowments $(L, W, K)$, without the demand side, the prices $(p, w_L, w_K)$, the food wage $(\omega)$, the outputs $(Y_1, Y_2)$, and the uniform rate of profit $(r)$ are determined from (1)-(7). Under the standard setting with $n$ goods, we require the equality of the demand and the supply in $n-1$ markets to close the system by the Walras' law. However, the above system with two goods does not require even the equality of the demand and the supply in one market. This is due to the use of the equality of profit rates of two capitals (the wage fund and the machine stock), which is absent from Ricardian two sector models with one capital (the wage fund only),[7] two-sector Hechscher-Ohlin-Samuelson models, and the standard neoclassical two sector growth models.[8] In the standard neoclassical static setting, where the factor endowments, $L$ and $K$, are given, the production side is composed of four equations, (3)-(6), and five unknowns, $p$, $w_L$, $w_K$, $Y_1$, and $Y_2$, so that, to close the system, the equality of the demand and the supply in one market is necessary. In the above Ricardian system with one additional state variable, $W$, three equations, (1), (2), and (7), and two unknowns, $r$ and $\omega$, are added to the neoclassical production side, so that the equality of the demand and the supply in one market is not necessary to close the static system.

Furthermore, since the outputs of the food and the machine are determined independently of the consumption patterns for them, if the consumption patterns are given, the net investments into the wage fund and the machine stock must be adjusted to keep the equality of the demand and the supply in the goods 1 and 2 markets, and the net investments must be passive. This implies Say's law of markets in the Ricardian system.[9]

7 For instance, Pasinetti (1960), Findlay (1974), Maneschi (1983), Costa (1985), Burgstaller (1986, 1989), and Uchiyama (2005).

8 For instance, Uzawa (1961) and Jones (1965).

9 See Morishima (1989, ch.7).

## 3 Existence and uniqueness of the momentary equilibrium: a graphical explanation

In this section, using simple figures, I show the conditions under which the static system, (1)-(7), has meaningful solutions for given factor endowments ($L$, $W$, $K$). First I consider the existence, the uniqueness, and the positive of the machine price, the factor prices, and the uniform rate of profit when both goods are produced, and second consider the conditions under which both goods are produced.

### 3.1 The momentary equilibrium under diversification in production

First suppose that both goods are produced. Then, to derive the relations between $p$, $w_L$, and $w_K$, from (3) and (4), we can use the familiar Stolper-Samuelson (1941) theorem: a rise of the relative price of the labour (machine) intensive good causes a rise (fall) of $w_L$ and a fall (rise) of $w_K$. This theorem plays important roles in the following analysis.

In Figure 1, the straight lines in the second quadrant represent equation (1) under $\omega = \omega_i (i = 1, 2, 3)$, where $\omega_3 > \omega_2 > \omega_1$. The curve $SS_W$ in the third quadrant represents the Stolper-Samuelson (1941) relation between $p$ and $w_L$ under $k_1 > k_2$, which is derived from (3) and (4): a rise of the price of the labour intensive good (the good 2, the machine), $p$, causes a rise of $w_L$ and a fall of $w_K$. Thus the curve $r(p, \omega_i)$ in the first quadrant represents the relation of $p$ and $r$ which satisfy (1), (3), and (4) under $k_1 > k_2$ and $\omega = \omega_i (i = 1, 2, 3)$. Apparently a rise of $\omega$ causes an outward shift and a decrease of the slope of the straight line (1) in the second quadrant, and thereby the curve $r(p, \omega_i)$ in the first quadrant also shifts outward and becomes gentler. Note that both the straight line (1) and the curve $r(p, \omega_i)$ converge to the vertical (horizontal) axis as $\omega_i \to 0 \ (+\infty)$.

In Figure 2 the 45° line in the second quadrant represents equation (2), and the curve $SS_{KP}$ in the third quadrant the Stolper-Samuelson relation between $p$ and $w_K/p$ under $k_1 > k_2$, which is derived from (3) and (4). Thus the curve $r(p)$ in the first quadrant represents the relation of $p$ and $r$ which satisfy (2)-(4) under $k_1 > k_2$.

We can now consider the determination of the machine price and the uniform rate of profit in the momentary equilibrium, where the factor endowments ($L$, $W$, $K$) are given. The curve $r(p, \omega_i)$ in panel (a) of Figure 3, which is derived in Figure 1, represents the combinations of $p$ and $r$ which satisfy (1), (3), and (4) under $k_1 > k_2$ and $\omega_i (i = 1, 2, 3)$, and the curve $r(p)$ in panel (a) of Figure 3, which is derived in Figure 2, the combinations of $p$ and $r$ which satisfy (2)-(4) under $k_1 > k_2$. Thus the point $E_i$ in panel (a) of Figure 3 represents unique combination of $r$ and $p$ which satisfy (1)-(4) under $k_1 > k_2$ and $\omega_i (_i = 1, 2, 3)$, namely the momentary equilibrium values of $r$ and $p$ under each $\omega$. Apparently higher $\omega$ corresponds to lower uniform rate of profit and higher machine price. Further the uniform rate of profit and the machine price in the above momentary equilibrium are always positive for any $\omega$. This is because the curve $SS_{KP}$ in the third quadrant of Figure 2, which represents the familiar Stolper-Samuelson relation, does not intersect the vertical and the horizontal axes and hence the curve $r(p)$ also does not intersect the vertical and the horizontal axes.

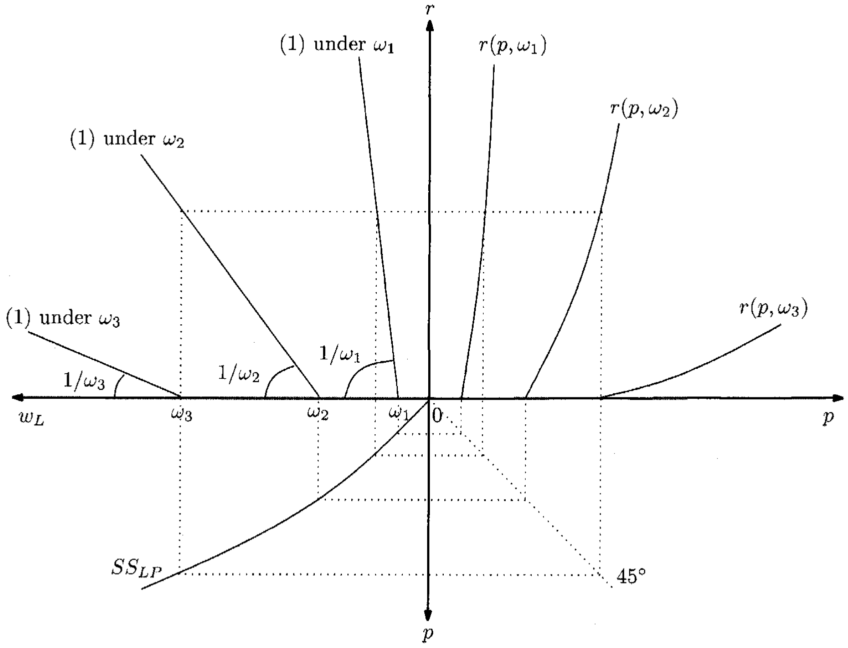


Figure 1: The relations between $p$, $r$ and $\omega$ under (1), (3), and (4): $k_1 > k_2$ and $\omega_3 > \omega_2 > \omega_1$ case

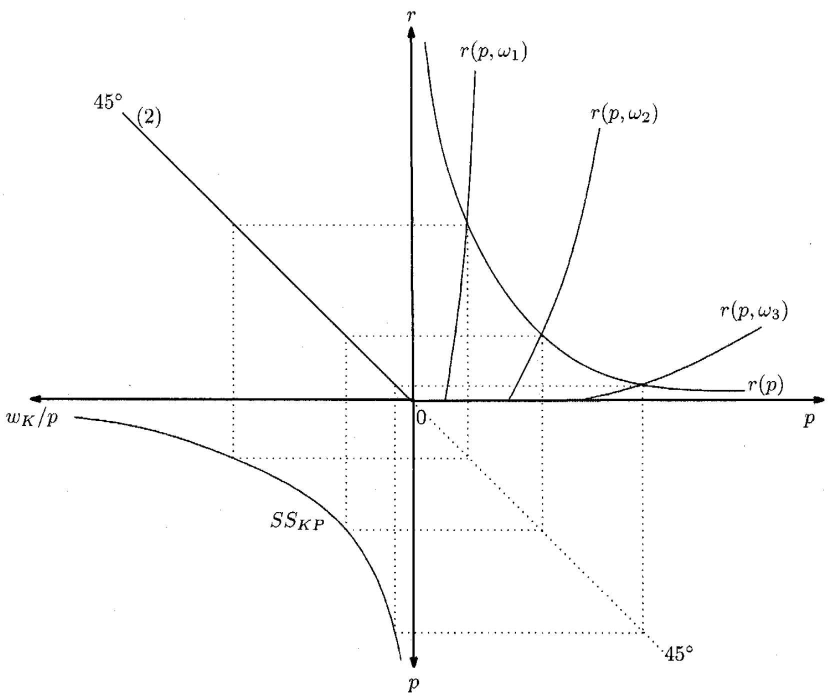


Figure 2: The relation between $p$ and $r$ under (2)-(4): $k_1 > k_2$ case

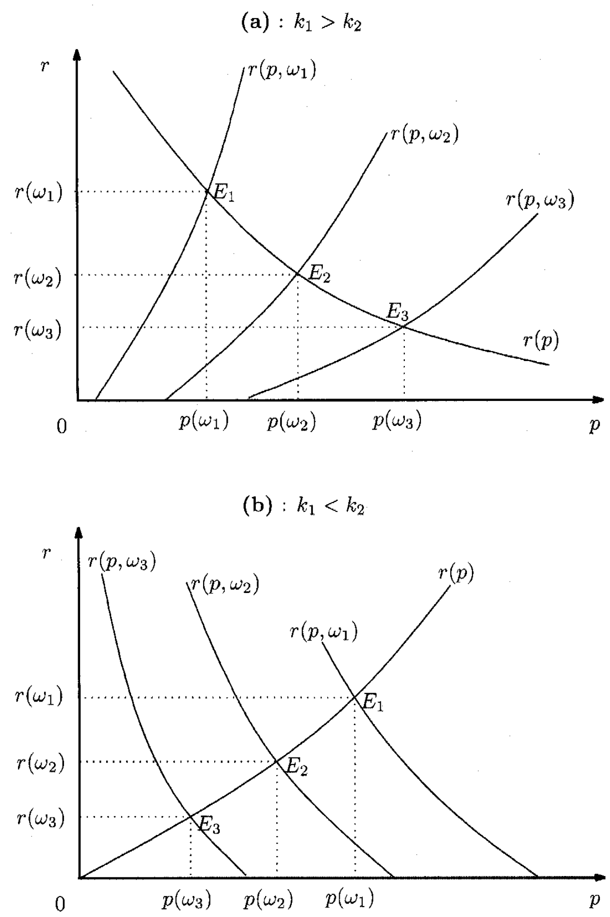


Figure 3: The determination of the relative price and the uniform rate of profit: $\omega_3 > \omega_2 > \omega_1$ case

On the other hand, when the good 2 sector is machine intensive, namely $k_1 < k_2$, a rise of the price of the machine intensive good (the good 2, the machine), $p$, causes a rise of $w_K$ and a fall of $w_L$ from the Stolper-Samuelson (1941) theorem. Further it is well known that absolute value of the increasing rate of $w_K$ is larger than one of $p$, which is called Jones's (1965) magnification effects. Thus the slopes of the curves $SS_W$ and $SS_{KP}$ in the third quadrant of Figures 1 and 2 are reversed, so that the slopes of the curves $r(p, \omega_i)$ and $r(p)$ in the first quadrant are also reversed. Hence, as shown in panel (b) of Figure 3, a rise of $\omega$ causes a fall of the uniform rate of profit and the machine price. Note that, also in this case, the unique existence and the positive of $p$, $w_L$, $w_K$, and $r$ are obtained for any $\omega$.

Furthermore, according to the Stolper-Samuelson (1941) theorem, a rise of $\omega$ always causes a rise of $w_L$ and a fall of $w_K$ because, from the above argument, a rise of $\omega$ causes a rise (fall) of $p$ if the good 2 sector is labour (machine) intensive.

Note that, as long as (1)-(4) hold, the above results are obtained even if only one good is produced. Thus we can summarize the above results as follows.

**Proposition 1 (The existence of momentary equilibrium)** *Suppose that both goods are produced or (1)-(4) hold.* [1] *There exist positive and unique $p$, $w_L$, $w_K$, and $r$ which satisfy*

*(1)-(4) for any $\omega > 0$.* [2] *A rise of $\omega$ always causes a fall of $r$ and $w_K$ and a rise of $w_L$ and the relative price of the labour intensive good.*[10]

I note that the above proposition holds also under constant $c_{ij}$, namely a single technique.

### 3.2 Conditions for diversification in production

Next consider conditions under which both goods are produced. Figure 4 illustrates the familiar relationships under $k_1 > k_2$ between $k_1$, $k_2$, and $w_L/w_K$ (in the second quadrant) and between $p$ and $w_L/w_K$ (in the third quadrant), where the curve $SS$ represents the Stolper-Samuelson relation under $k_1 > k_2$ between $p$ and $w_L/w_K$. The curve $p(\omega)$ in the forth quadrant of the figure represents the relationship under $k_1 > k_2$ between $\omega$ and $p$ (the relative price of the labour intensive good) which satisfy (1)-(4), namely [2] in Proposition 1. Similarly, Figure 5 illustrates the case under $k_1 < k_2$.

From Figures 4 and 5, if both goods are produced under $\omega_1$, then

$$\max[k_1(\omega_1), k_2(\omega_1)] > k > \min[k_1(\omega_1), k_2(\omega_1)] \tag{12}$$

must hold, where $k \equiv K/L$ is the machine/labour ratio in the economy. On the other hand, if $p_{\max}(k) > p > p_{\min}(k)$ holds, both goods are produced under $k$. Hence if (12) holds, then $p_{\max}(k) > p(\omega_1) > p_{\min}(k)$ holds, so that both goods are produced under $\omega_1$. Thus both goods are produced under $\omega_1$ if and only if (12) holds. Since $\omega_1$ is arbitrary, both goods are produced under (1)-(7) if and only if the economy is located between the curves $k_1(\omega)$ and $k_2(\omega)$ in the first quadrant of Figures 4 and 5. Note that, if $k = k_i(\omega)(i = 1,2)$, to satisfy (5) and (6), only good $i$ must produced under $p(\omega)$ which satisfies (1)-(4).

On the other hand, from the above argument, the economy must specialize in production of either good if $k > \max[k_1(\omega), k_2(\omega)]$, or if $\min[k_1(\omega), k_2(\omega)] > k$. Which good is produced? We can show that only good 1 is produced if $k > \max[k_1(\omega), k_2(\omega)]$ while only good 2 is produced if $\min[k_1(\omega), k_2(\omega)] > k$.[11]

Thus we can summarize the above results as follows.[12]

**Proposition 2 (Conditions for the diversification in production)** [1] *Both goods are produced if and only if the economy is located between the curves $k_1(\omega)$ and $k_2(\omega)$, and only good $i$ is produced if the economy is located on the curve $k_i(\omega)(i = 1,2)$.* [2] *Only good 1 is produced if $k > \max[k_1(\omega), k_2(\omega)]$, and only good 2 is produced if $\min[k_1(\omega), k_2(\omega)] > k$.* [3] *A rise of the market food wage causes the switching of techniques toward one using the machine more intensively in both sectors if the economy is located between the curves $k_1(\omega)$ and $k_2(\omega)$.*

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10 See Appendix 2 for the formal proof of [2] in the proposition.

11 See Appendix 1.

12 Note that, from Figures 4 and 5, a rise of $\omega$ causes a rise of both $k_1(\omega)$ and $k_2(\omega)$, namely the switching of techniques toward one using the machine more intensively in both sectors.

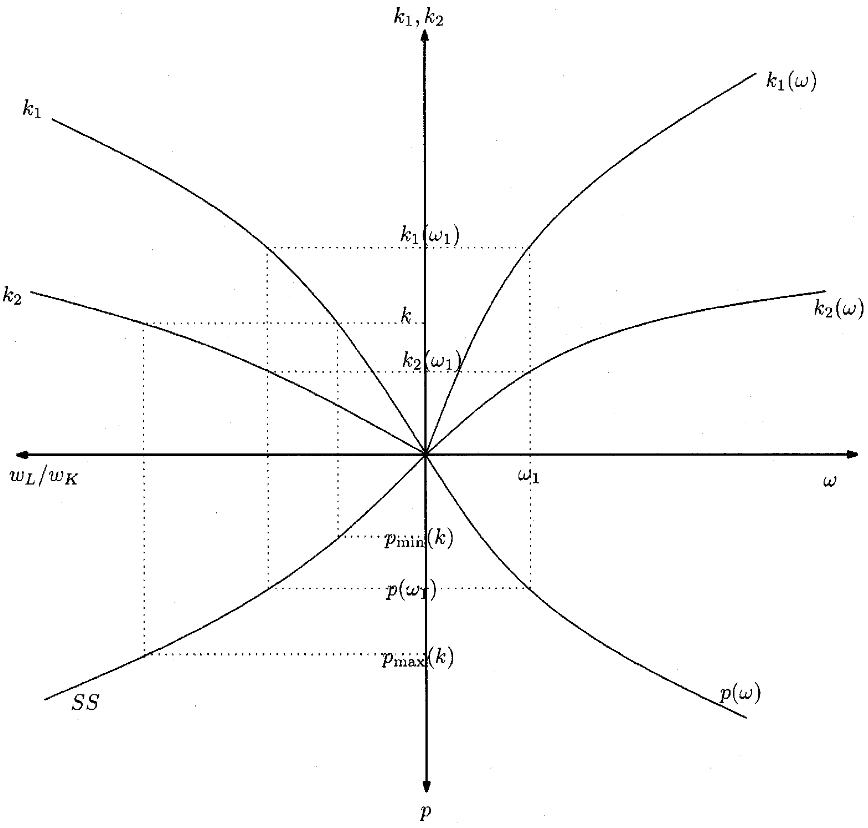


Figure 4: The conditions for diversification in production: $k_1 > k_2$ case

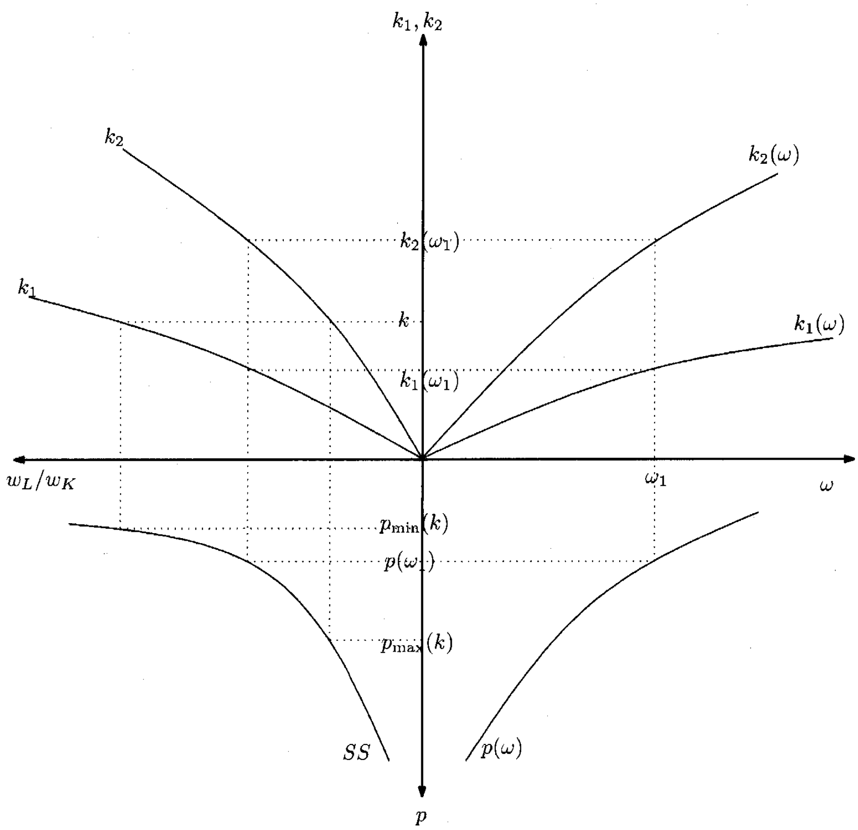


Figure 5: The conditions for diversification in production: $k_1 < k_2$ case

Note that the uniqueness and the positive of $p$, $w_L$, $w_K$, and $r$ are satisfied even if $k > \max[k_1(\omega), k_2(\omega)]$ or $\min[k_1(\omega), k_2(\omega)] > k$.[13]

From [2] in Proposition 1 and [3] in Proposition 2, we can conclude that, under diversification in production, a rise of $\omega$ always causes the switching of techniques toward one using the machine more intensively in both sectors even if the rise of $\omega$ causes a rise of the machine price. This is the answer to our first question in Section 1.

## 4 Advanced food wage, factor endowments, and outputs

In this section, I consider the relations between the advanced food wage, the factor endowments, and the outputs.

From (5) and (6), we have

$$Y_1 = c_{2L}(w_L, w_K)L(k_2 - k)/|c|,$$

$$Y_2 = c_{1L}(w_L, w_K)L(k - k_1)/|c|,$$

or

$$y_1 \equiv Y_1/L = c_{2L}(w_L, w_K)(k_2 - k)/|c|, \tag{13}$$

$$y_2 \equiv Y_2/L = c_{1L}(w_L, w_K)(k - k_1)/|c|, \tag{14}$$

where

$$\begin{aligned} |c| &\equiv c_{1L}(w_L, w_K)c_{2K}(w_L, w_K) - c_{2L}(w_L, w_K)c_{1K}(w_L, w_K) \\ &= c_{1L}(w_L, w_K)c_{2L}(w_L, w_K)(k_2 - k_1). \end{aligned}$$

Note that, from the argument in the previous section, both $w_L$ and $w_K$ are functions of only $\omega$ as long as both goods are produced. Thus both $y_1$ and $y_2$ are functions of only $\omega$ and $k$ as long as both goods are produced: $y_i = y_i(\omega, k)(i=1,2)$.[14] Then

$$y_{2\omega},\ y_{1k} > (<) 0 > (<) y_{1\omega}, y_{2k} \text{ as } k_1 > (<) k_2 \tag{15}$$

holds, where $y_{ij} \equiv \partial y_i / \partial_j (i=1, 2, j=\omega, k)$.[15]

Thus from (15) we have the following results.

**Proposition 3 (Food wage, factor endowments ratio, and outputs)** *Suppose that both goods are produced.* [1] *For given k, a rise of the advanced food wage, $\omega$, causes an increase of the output per labour of the labour-intensive good and a decrease of the output per labour of the*

13 See Appendix 1.

14 If the economy specializes in the good $i(i=1, 2)$, $y_i = F_i(1, k)$, so that $y_i$ is a function of only $k$.

15 The signs of $y_{1k}$ and $y_{2k}$ are easily obtained by the direct calculation. See Appendix 2 for the signs of $y_{1\omega}$ and $y_{2\omega}$.

*machine-intensive good.* [2] *For given $\omega$, a rise of the machine/labour ratio in the economy, $k$, causes a decrease of the output per labour of the labour-intensive good and an increase of the output per labour of the machine-intensive good.*

From [1] in the above proposition, we can conclude that, for given $L$ and $K$, higher $\omega$ due to larger $W$ corresponds to smaller $Y_1$, which means smaller $W$ in the next period, if and only if the food sector is machine intensive. This is the answer to our second question in Section 1.

## 5 Dynamics, the steady state, and the stability

In this section, I consider the dynamic interactions between the mechanization and the advanced food wage.

From the argument in the previous section, both $y_1$ and $y_2$ are the functions of only $\omega$ and $k$ as long as both goods are produced. On the other hand, we can show that the intertemporal changes of $\omega$ and $k$ depend on $\omega$, $k$, $y_1$, and $y_2$: from (7)-(9) and (11),

$$\dot{\omega}=(\dot{W}/W-\dot{L}/L)\omega=y_1-[1+g(\omega,\ \bar{\omega})]\omega, \tag{16}$$

$$\dot{k}=(\dot{K}/K-\dot{L}/L)k=y_2-g(\omega,\ \bar{\omega})k. \tag{17}$$

Note that (16) and (17) hold also when only one good is produced. Thus, as seen later, if the initial values of $\omega$ and $k$ are given, we can characterize fully the intertemporal behavior of $\omega$ and $k$, and thereby understand the intertemporal tendencies of the prices, the uniform rate of profit, the switching of techniques, and the patterns of production in the above Ricardian system from the argument in Section 3.

Define the steady state as the pair of $\omega$ and $k$ which satisfy $\dot{\omega}=\dot{k}=0$, and denote the steady state values of $\omega$ and $k$ as $\omega_S$ and $k_S$ respectively. Then we can easily show that there is no steady state under specialization.[16] Hence, in the following, I deal with the steady state under diversification in production.

The stability conditions of the steady state under diversification in production are, from the linear approximation of (16) and (17) around the steady state and $g=y_1/\omega-1=y_2/k$ ($\Leftrightarrow \dot{\omega}=\dot{k}=0$ at the steady state),

$$(y_{1\omega}-y_1/\omega-g_1\omega)+(y_{2k}-y_2/k)<0, \tag{18}$$

$$(y_{1\omega}-y_1/\omega-g_1\omega)(y_{2k}-y_2/k)-y_{1k}(y_{2\omega}-g_1k)>0. \tag{19}$$

---

16 If in the steady state the economy specializes in the good 1, then $g=0$ from (17) and $\dot{k}=y_2=0$, so that $y_1=\omega$ from (16) and $\dot{\omega}=g=0$, which implies that the steady state value of the uniform rate of profit is zero. However, from the argument in Section 3, the uniform rate of profit is positive even under specialization, which is a contradiction. If in the steady state the economy specializes in the good 2, then $g=-1$ from (16) and $\dot{\omega}=y_1=0$, so that $y_2=-k$ from (16), $\dot{k}=0$, and $g=-1$, which is a contradiction.

Thus we can show that it is necessary for the stability of the steady state that the good 1 sector is machine intensive: $k_1 > k_2$.[17] On the other hand, if the good 1 sector is machine intensive, (18) is always satisfied since $y_{1\omega}<0$ and $y_{2k}<0$ hold from (15). Further, since $y_{1k}>0$ holds, (19) is also satisfied if the change rate of the labour responds to the change of the food wage more sensitively than the change rate of the machine stock: $g_1 > y_{2\omega}/k = \partial(\dot{K}/K)/\partial\omega$.

The above arguments can be summarized as follows.

**Proposition 4 (The stability conditions of the steady state)**

[1] *The steady state is dynamically stable if and only if the good 1, the food, sector is machine intensive ($k_1 > k_2$) and (19) holds.* [2] *If the good 1, the food, sector is machine intensive ($k_1 > k_2$), and if the change rate of the labour responds to the change of the food wage more sensitively than the change rate of the machine stock ($g_1 > y_{2\omega}/k$), the steady state is dynamically stable.*

Now consider the dynamic behavior of the Ricardian machinery system by the phase diagram in $\omega$ - $k$ plane. In the following, I deal with the stable steady state case: I assume $k_1 > k_2$ and $g_1 > y_2\omega/k$ ([2] in Proposition 4). Thus Figure 6, based on the first quadrant of Figure 4, illustrates the dynamics of the system.

First consider the locus of $\dot{k}=0$. It has the negative slope when the good 2 is produced.[18] Note that $\dot{k}=0$ holds at the point $A$ and over the point since $y_2=0$ and $g=0$ hold, and that the locus of $\dot{k}=0$ does not intersect the horizontal axis.[19] Apparently $\dot{k}>0$ holds on the left side of the locus of $\dot{k}=0$, and $\dot{k}<0$ on the right side of the locus.[20]

Next consider the locus of $\dot{\omega}=0$. It has the positive slope when the good 1 is produced.[21]

17 If $k_1 < k_2$, then $y_{1k}<0$, $y_{2\omega}<0$, and $y_{2k}-y_2/k=c_{1K}/|c|>0$ hold from (14) and (15), so that $y_{1\omega}-g-g_1-1>0$ must hold to satisfy (19) while $y_{1\omega}-g-g_1-1<0$ must hold to satisfy (18). Hence (18) and (19) cannot hold at the same time if $k_1<k_2$.

18 As long as both goods are produced, from (15) and (17),

$$\left.\frac{dk}{d\omega}\right|_{\dot{k}=0} = -\frac{y_{2\omega}-g_1k}{y_{2k}-y_2/k}<0,$$

while, under specialization in the good 2,

$$\left.\frac{dk}{d\omega}\right|_{\dot{k}=0} = \frac{g_1k^2}{[\partial F_2(1,k)/\partial k - F_2(1,k)/k]-y_2}<0.$$

Note that, under specialization in the good 2, $y_2=F_2(1,k)$, and that $\partial F_2(1,k)/\partial k - F_2(1,k)/k<0$ holds from the concavity of $F_2$.

19 Under specialization in the good 2, $\dot{k}=[F_2(1,k)/k-g(\omega,\bar{\omega})]$ k, while $\lim_{k\to 0} F_2(1,k)/k=+\infty$ by the L'Hôpital's rule and Inada condition. Thus, under any $\omega$, $\dot{k}>0$ for sufficiently small $k$.

20 Define $\eta \equiv y_2-gk(=\dot{k})$. If both goods are produced, $\partial\eta/\partial\omega = y_{2\omega}-g_1k<0$ by the assumption. If the economy specializes in either good, $\partial\eta/\partial\omega=-g_1k<0$.

21 As long as both goods are produced, from (15) and (16),

$$\left.\frac{dk}{d\omega}\right|_{\dot{\omega}=0} = -\frac{y_{1\omega}-y_1/\omega-g_1\omega}{y_{1k}}>0,$$

while, under specialization in the good 1,

$$\left.\frac{dk}{d\omega}\right|_{\dot{\omega}=0} = \frac{y_1/\omega+g_1\omega}{\partial F_1(1,k)/\partial k}>0.$$

Note that, under specialization in the good 1, $y_1=F_1(1,k)$.

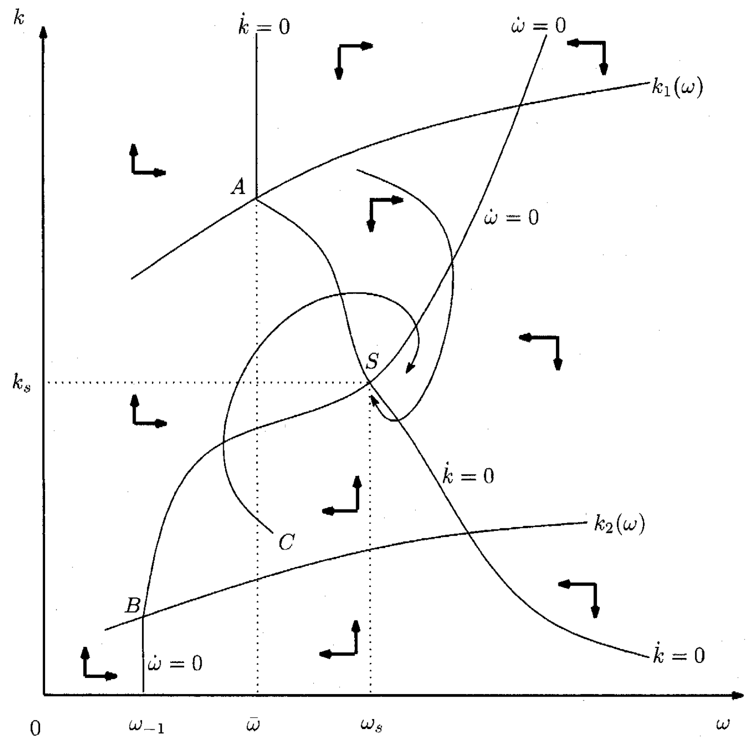


Figure 6: Dynamics of Ricardian machinery system

Define $\omega_{-1}$ as $\omega$ which satisfies $g(\omega, \bar{\omega})=-1$. Then $\dot{\omega}=0$ holds at the point $B$ and under the point since $y_1=0$ and $g=-1$ hold.[22] Apparently $\dot{\omega}>0$ holds for $\omega<\omega_{-1}$ by $1+g<0$. On the other hand, $\dot{\omega}>0$ holds above the locus of $\dot{\omega}=0$, and $\dot{\omega}<0$ below the locus.[23]

Thus the loci of $\dot{k}=0$ and $\dot{\omega}=0$ intersect only at the point $S$ located between the curves $k_1(\omega)$ and $k_2(\omega)$. The point $S$ in Figure 6 is the unique and stable steady state. Apparently $\omega_s>\bar{\omega}$ holds, so that, at the steady state, the labour, the wage fund, and the machine stock grow at the positive constant rate which is equal to the uniform rate of profit.[24] On the other hand, an exogenous rise of the subsistence wage causes the rightward shifts of the loci of $\dot{k}=0$ and $\dot{\omega}=0$,[25] so that the steady state food wage, $\omega_s$, rises, and thereby the uniform rate of profit at the steady state, which is equal to the steady state growth rates of $L$, $W$, and $K$, falls.

Thus we have the following results.

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22 If $g(0, \bar{\omega})\geq-1$, there is no $\omega_{-1}$. Then the locus of $\dot{\omega}=0$ is located between the curves $k_1(\omega)$ and $k_2(\omega)$. Proof: If $g(0, \bar{\omega})\geq-1$, $-(1+g)\omega<0$ holds for $\omega<\bar{\omega}$, so that $\dot{\omega}=-(1+g)\omega<0$ holds on and below the curve $k_2(\omega)$ for $\omega<\bar{\omega}$, while $\dot{\omega}=(y_1-\omega)-g\omega>0$ on and above the curve $k_1(\omega)$ for $\omega<\bar{\omega}$ since $y_1-\omega>0$ from $r>0$ under specialization. Thus, by $y_{1k}>0$, for $\omega<\bar{\omega}$, the locus of $\dot{\omega}=0$ is uniquely located between the curves $k_1(\omega)$ and $k_2(\omega)$. Q.E.D.

23 Define $\rho\equiv y_1-[1+g(\omega, \bar{\omega})]\ \omega(=\dot{\omega})$. If the good 1 is produced, $\partial\rho/\partial k=y_{1k}>0$, while if the economy specializes in the good 2, $\partial\rho/\partial k=0$.

24 In the steady state, $\dot{\omega}=\dot{k}=0 \Leftrightarrow \dot{W}/W=\dot{K}/K=g>0$, and $g=r$ from (10) and $\dot{W}/W=\dot{K}/K=g$.

25 For given $k$, $d\omega/d\bar{\omega}|\eta=0=g_2k/(y_{2\omega}-g_1k)>0$ (where $y_{2\omega}=0$ if either good is not produced), which implies the rightward shift of the locus of $\dot{k}=0$ by the rise of $\bar{\omega}$. For given $k$, $d\omega/d\bar{\omega}|_{\rho=0}=g_2\omega/(y_{1\omega}-y_1/\omega-g_1\omega)>0$ (where $y_{1\omega}=0$ if either good is not produced), which implies the rightward shift of the locus of $\dot{\omega}=0$ by the rise of $\bar{\omega}$.

**Proposition 5 (Properties of the steady state)** *Suppose that the food sector is machine intensive, and that the change rate of the labour responds to the change of the food wage more sensitively than the change rate of the machine stock.* [1] *There is the unique and stable steady state.* [2] *At the steady state, the labour, the wage fund, and the machine stock grow at the positive constant rate which is equal to the uniform rate of profit.* [3] *An exogenous rise of the subsistence wage causes a rise of the steady state food wage and a fall of the uniform rate of profit and the growth rates of $L$, $W$, and $K$ at the steady state.*

From [1] in Proposition 4, there is no stable steady state if the good 1 sector is labour intensive, and the good 1 sector must be machine intensive for the dynamic stability of the wage and the technique in the long run equilibrium (steady state) to be satisfied. Further, from [1] in Proposition 5, the wage and the technique in the long run equilibrium is unique if the food sector is machine intensive, and if the change rate of the labour responds to the change of the food wage more sensitively than the change rate of the machine stock. This is the answer to our third question in Section 1.

Now we can derive some dynamic properties of the above Ricardian machinery system from Figures 6. First the advanced food wage, $\omega$, continues to rise as long as the economy locates between the curve $k_1(\omega)$ and the locus of $\dot{\omega}=0$, so that the uniform rate of profit keeps falling from [2] in Proposition 1 while the machine/labour ratios in both sectors keep rising from [3] in Proposition 2.

Second the advanced food wage continues to fall as long as the economy locates between the curve $k_2(\omega)$ and the locus of $\dot{\omega}=0$, so that the uniform rate of profit keeps rising from [2] in Proposition 1 while the machine/labour ratios in both sectors keep falling from [3] in Proposition 2.

Third the phase where $\omega$ continues to rise (hence the uniform rate of profit keeps falling while the machine/labour ratios in both sectors keep rising) and the converse one appear alternately on any dynamic path toward the steady state.

Fourth a rise of the advanced food wage and the regression of the mechanization (a fall of the machine/labour ratio in the economy) can bring about at the same time. Thus a rise of labour price does not always promote the mechanization in the economy.

Fifth, if the initial values of $\omega$ and $k$ are represented by the point such as $C$, then $\omega$ on the dynamic path toward the steady state continues to be below the initial value in the short run, but continues to exceed it in the long run. Thus if the initial value of $\omega$ is the subsistence level, the wage level version of Ricardo's argument on the effects of mechanization will be valid when the initial machine/labour ratio in the economy is comparatively small (below the locus of $\dot{\omega}=0$).

The above is the answer to our forth question in Section 1.

## 6 Conclusions

This paper has developed a Ricardian machinery two sector model composed of the food and the machinery sectors with NIM mechanism. Using the model, the followings have been shown. A rise of the advanced wage always causes the use of the technique using the machine more intensively even when the rise of the advanced wage causes a rise of the machine price. Given the labour and the machine stock, a rise of the advanced wage causes a decrease of the output of the food if and only if the food sector is machine intensive. It is necessary for the dynamic stability of the advanced wage and the technique in the long run equilibrium (steady state) that the food sector is machine intensive. There can be the dynamic path to the steady state on which, from the viewpoint of the level of the wage, Ricardo's argument on the effects of mechanization holds. I also note that, even if the machine is produced by the labour alone as in Ricardo's numerical example and the other Ricardian machinery models (namely, with $k_2 = c_{2K} = 0$, the curve $k_2(\omega)$ in Figures 4 and 6 coincides with the horizontal axis), the above results still hold.

## Appendix 1: Patterns of specialization

In this appendix, I consider the patterns of specialization when $k > \max[k_1(\omega), k_2(\omega)]$ or $k < \min[k_1(\omega), k_2(\omega)]$.

From the argument in Section 3, $p$, $w_L$, $w_K$, and $r$ which satisfy (1)-(4) are functions of $\omega$. Let $m(\omega)$, $m^*$, and $m^{**}$ denote variable $m(=p, w_L, w_K, r, c_i, c_{ij})$ which satisfies (1)-(4) under $\omega$, one when only good 1 is produced, and one when only good 2 is produced respectively. Suppose that $\omega$ is given.

When only good 1 is produced, $w_L = F_1(1, k) - k\partial F_1(1, k)/\partial k$ holds, so that $w_L$ is a function of only $k$: $w_L^* = w_L^*(k)$ where $dw_L^*/dk = -k\partial^2 F_1/\partial k^2 > 0$. Similarly, $w_K = \partial F_1(1, k)/\partial k$ also holds, so that $w_K$ is a function of only $k$: $w_K^* = w_K^*(k)$ where $dw_K^*/dk = \partial^2 F_1/\partial k^2 < 0$. Further, $r$ is also a function of only $k$ from (1): $r^* = r^*(k)$ where $dr^*/dk = (dw_L^*/dk)/\omega = -(\mathrm{k}\partial^2 F_1/\partial k^2)/\omega > 0$, and $p$ is also so from (2): $p^* = p^*(k)$ where $dp^*/dk = (dw_K^*/dk - p^* dr^*/dk)/r^* < 0$. Note that $1 = c_1[w_L^*(k), w_K^*(k)]$ holds.

When only good 2 is produced, $r^{**} = \partial F_2(1, k)/\partial k$ holds from (2) and $w_K^{**} = p^{**}\partial F_2(1, k)/\partial k$, so that $r$ is a function of only $k$: $r^{**} = r^{**}(k)$ where $dr^{**}/dk = \partial^2 F_2/\partial k^2 < 0$. Hence $w_L$ is also a function of only $k$ from (1): $w_L^{**} = w_L^{**}(k)$ where $dw_L^{**}/dk = \omega dr^{**}/dk < 0$. Thereby $p$ is also a function of only $k$ from $w_L^{**} = p^{**}[F_2(1, k) - k\partial F_2(1, k)/\partial k]$: $p^{**} = p^{**}(k)$ where $dp^{**}/dk = (dw_L^{**}/dk + p^{**}k\partial^2 F_2/\partial k^2)/[F_2(1, k) - k\partial F_2(1, k)/\partial k] < 0$. Finally $w_K$ is also a function of only $k$ from (2): $w_K^{**} = w_K^{**}(k)$ where $dw_K^{**}/dk = p^{**}dr^{**}/dk + r^{**}dp^{**}/dk < 0$. Note that $p^{**}(k) = c_2[w_L^{**}(k), w_K^{**}(k)]$ holds.

**Specialization pattern under** $k > \max[k_1(\omega), k_2(\omega)]$

Which good is produced if $k > \max[k_1(\omega), k_2(\omega)]$ ?

For $k = k_2(\omega)$, (1)-(4) and $Y_2 > 0 = Y_1$ hold at the same time, so that $w_L(\omega) = w_L^{**}(k)$ and $w_K(\omega_1) = w_K^{**}(k)$ hold. Hence, for $k > \max[k_1(\omega), k_2(\omega)]$, $w_L(\omega) > w_L^{**}(k)$ and $w_K(\omega) > w_K^{**}(k)$ hold from $dw_L^{**}/dk < 0$ and $dw_K^{**}/dk < 0$. Thus, for $k > \max[k_1(\omega), k_2(\omega)]$, $1 > c_1[w_L^{**}(k), w_K^{**}(k)]$ and $p^{**}(k) = c_2[w_L^{**}(k), w_K^{**}(k)]$ hold from $1 = c_1[w_L(\omega), w_K(\omega)]$. This implies that the specialization in the good 2 under $k>\max[k_1(\omega), k_2(\omega)]$ can never be carried out, so that only good 1 is produced if $k>\max[k_1(\omega), k_2(\omega)]$.

On the other hand, if only good 1 is produced, then $w_L^*(k)=F_2(1, k)-k\partial F_2(1, k)/\partial k>0$, $w_K^*(k)=\partial F_2(1, k)/\partial k>0$, $r^*(k)=w_L^*(k)/\omega_1-1>0$ (from (1)), and $p^*(k)=w_K^*(k)/r^*(k)>0$ (from (2)) must hold. The positive of $w_L^*(k)$ and $w_K^*(k)$ is evident. The positive of $r^*(k)$ follows from $r^*(k)\geq r(\omega)>0$ for $k=\max[k_1(\omega), k_2(\omega)]$ and $dr^*/dk>0$, so that the positive of $p^*(k)$ is also evident.

**Specialization pattern under $k<\min[k_1(\omega), k_2(\omega)]$**

Which good is produced if $\min[k_1(\omega), k_2(\omega)]>k$?

For $k=k_1(\omega)$, (1)-(4) and $Y_1>0=Y_2$ hold at the same time, so that $p(\omega)=p^*(k)$, $w_L(\omega)=w_L^*(k)$, and $w_K(\omega)=w_K^*(k)$. Define $\psi(k)\equiv p^*(k)-c_2^*[w_L^*(k), w_K^*(k)]$. Since $\psi[k_1(\omega)]=p(\omega)-c_2[w_L(\omega), w_K(\omega)]=0$ and

$$\frac{d\psi}{\mathrm{dk}}=(p^*-w_K^*c_{2K}^*)\frac{dw_K^*/dk}{\mathrm{p^*r^*}}-\frac{p^*dr^*/dk}{\mathrm{r^*}}-c_{2L}^*\frac{dw_L^*}{dk},$$

$d\psi/dk<0$ holds under $k=k_1(\omega)$. Hence, under $k=k_1(\omega)-dk$, $p^*>c_2^*=w_L^*c_{2L}^*+w_K^*c_{2K}^*$ and $d\psi/dk<0$ hold. Thereby, also under $k=k_1(\omega)-2dk$, $p^*>c_2^*=w_L^*c_{2L}^*+w_K^*c_{2K}^*$ and $d\psi/dk<0$ hold. Repeating this procedure, $p^*>c_2^*$ holds under any $k<\min[k_1(\omega), k_2(\omega)]$. Thus when only good 1 is produced, there is no $k$ which satisfies $\psi(k)\leq 0$ and $\min[k_1(\omega), k_2(\omega)]>k$, while $1=c_1^*$ for $\forall k<\min[k_1(\omega), k_2(\omega)]$. This implies that the specialization in the good 1 under $k<\min[k_1(\omega), k_2(\omega)]$ can never be carried out, so that only good 2 is produced if $\min[k_1(\omega), k_2(\omega)]>k$.

On the other hand, if only good 2 is produced, then $w_L^{**}=(1+r^{**})\omega_1>0$, $w_K^{**}=p^{**}\partial F_2(1, k)/\partial k>0$, $r^{**}=\partial F_2(1, k)/\partial k>0$ (from (2)), and $p^{**}=w_L^{**}/[F_2(1, k)-k\partial F_2(1, k)/\partial k]>0$ must hold. The positive of $r^{**}$ is evident, and thereby $w_L^{**}$ is also positive. Hence $p^{**}$ and $w_K^{**}$ are also positive.

Thus, since $\omega$ is arbitrary in the above argument, we have [2] in Proposition 2.

## Appendix 2: The derivation of $y_{1\omega}$ and $y_{2\omega}$

In this appendix, using Jones's (1965) "hat" comparative statics analysis, I derive $y_{1\omega}$ and $y_{2\omega}$. Let $\hat{m}\equiv dm/m$, $\theta_{1j}\equiv w_jc_{1j}$, and $\theta_{2j}\equiv w_jc_{2j}/p(j=L, K)$ denote the change rate of any variable $m$, the share of the factor $j$ in the good 1 sector, and the share of the factor $j$ in the good 2 sector respectively.

Totally differentiating (1) and (2),

$$dr=[dw_L-(1+r)d\omega]/\omega=(1+r)(\hat{w}_L-\hat{\omega}), \tag{20}$$

$$dr=(dw_K-rdp)/p=r(\hat{w}_K-\hat{p}), \tag{21}$$

so that from these equations we have

$$\hat{p} = \hat{w}_K - (1+r)(\hat{w}_L - \hat{\omega})/r. \tag{22}$$

Totally differentiating (3) and (4), using the $\theta_{ij}$ notation, and taking into consideration the first order conditions for the cost minimization, namely

$$w_L dc_{1L} + w_K dc_{1K} = \theta_{1L}\hat{c}_{1L} + \theta_{1K}\hat{c}_{1K} = 0, \tag{23}$$

$$w_L dc_{2K} + w_K dc_{2K} = \theta_{2L}\hat{c}_{2L} + \theta_{2K}\hat{c}_{2K} = 0, \tag{24}$$

we have

$$0 = \theta_{1L}\hat{w}_L + \theta_{1K}\hat{w}_K,$$

$$\hat{p} = \theta_{2L}\hat{w}_L + \theta_{2K}\hat{w}_L,$$

so that we have

$$\begin{bmatrix} \hat{w}_L \\ \hat{w}_K \end{bmatrix} = \frac{1}{|\theta|}\begin{bmatrix} \theta_{2K} & -\theta_{1K} \\ -\theta_{2L} & \theta_{1L} \end{bmatrix}\begin{bmatrix} O \\ \hat{p} \end{bmatrix}, \tag{25}$$

where

$$|\theta| \equiv \theta_{1L}\theta_{2K} - \theta_{1K}\theta_{2L} = \theta_{1L}\theta_{2L}(k_2 - k_1)w_K/w_L.$$

Substituting (25) into (22), we have

$$\hat{p} = -[|\theta|(1+r)/A]\hat{\omega}, \tag{26}$$

where $A \equiv \theta_{2L}r + \theta_{1K}(1+r)$. Substituting (26) into (25), we have

$$\hat{w}_L = [\theta_{1K}(1+r)/A]\hat{\omega}, \tag{27}$$

$$\hat{w}_K = -[\theta_{1L}(1+r)/A]\ \hat{\omega}. \tag{28}$$

Substituting (26)-(28) into (20) or (21), we have

$$dr = -[\theta_{2L}(1+r)r/A]\hat{\omega}. \tag{29}$$

Note that (26)-(29) directly prove [2] in Proposition 1.

From (13) and (14),

$$y_{1\omega} = [(dc_{2K}/d\omega - kdc_{2L}/d\omega)|c| - (c_{2K} - kc_{2L})d|c|/d\omega]/(|c|^2), \tag{30}$$

$$y_{2\omega} = [(kdc_{1L}/d\omega - dc_{1K}/d\omega)|c| - (kc_{1L} - c_{1K})d|c|/d\omega]/(|c|^2), \tag{31}$$

where

$$d|c|/d\omega = c_{2K}dc_{1L}/d\omega + c_{1L}dc_{2K}/d\omega - c_{1K}dc_{2L}/d\omega - c_{2L}dc_{1K}/d\omega.$$

Define $\sigma_i \equiv (\hat{c}_{iK} - \hat{c}_{iL})/(\hat{w}_L - \hat{w}_K)(i=1, 2)$. Note that $\sigma_i$ is the elasticity of substitution of the good $i$ sector ($i$=1,2), and that it is positive in the standard neoclassical production functions with the smooth isoquants convex to the origin. From the definition of $\sigma_i$, (23), and (24) we have

$$\hat{c}_{iL}=-\theta_{iK}\sigma_i(\hat{w}_L-\hat{w}_K),\ (i=1,2), \tag{32}$$

$$\hat{c}_{iK}=\theta_{iL}\sigma_i(\hat{w}_L-\hat{w}_K),\ (i=1,2). \tag{33}$$

Substituting (27) and (28) into (32) and (33),

$$dc_{iL}/d\omega=-\theta_{iK}c_{iL}(1+r)\sigma_i/(A\omega),\ (i=1,2), \tag{34}$$

$$dc_{iK}/d\omega=\theta_{iL}c_{iK}(1+_r)\sigma_i/(A\omega).\ (i=1,2). \tag{35}$$

Substituting (34) and (35) into (30) and (31), we have

$$y_{1\omega}=[c_{1L}(k_1-k)\alpha+c_{2L}(k_2-k)\beta]/(|c|^2), \tag{36}$$

$$y_{2\omega}=-[c_{1L}(k_1-k)\alpha+c_{2L}(k_2-k)\beta]/(p|c|^2), \tag{37}$$

where

$$\alpha\equiv-(1+r)c_{2L}c_{2K}\sigma_2/(A\omega)<0,$$

$$\beta\equiv(1+r)c_{1L}c_{1K}p\sigma_1/(A\omega)>0.$$

Thus we have the signs of $y_{1\omega}$ and $y_{2\omega}$ in (15).

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