経済と経営 30－2 (1999. 9)

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Trap of a *Ceteris Paribus* Clause in Inductive Argument
— An Essay on Theory of Confirmation (No.1) —

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ABSTRACT

Modern inductive sciences, e.g., mathematical theory of probability, statistics and economics, are now in the face of a *lack of reality*. This paper argues the logical problem on a variety of *petitio principii* arisen from a *ceteris paribus* clause available to usual inference. The problem concretely is that on the validity of a *ceteris paribus* clause to confirmation of hypothesis from the point of view of epistemology, not from that of mathematics. First, we tentatively define *conditional probability-relation* and *argument-condition probability* to solve the problem. Secondly, we specify the logical form of a *ceteris paribus* clause in inductive argument for confirmation of hypothesis. Logical analysis on the validity of a *ceteris paribus* clause to inductive argument for confirmation of hypothesis is put into practice from the point of view of epistemology as the result. In conclusion, trap of a *ceteris paribus* clause in inductive

argument for confirmation of hypothesis is clarified for the construction of new fundamental theory of confirmation of hypothesis.

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1 INTRODUCTION

Modern inductive sciences, e.g., mathematical theory of probability, statistics and economics, are now in the face of a lack of reality which means that mathematical principle in confirmation of hypothesis or scientific theory has extremely deviated from the original logic[1], from the point of view of the history of theory of probability in Britain. Many unverified premises are smuggled into inductive argument for confirmation of hypothesis, without approval, as the result of the deviation. This just means the ordinary form, without philosophical consideration, of the theory of confirmation of hypothesis.

We here argue an epistemological problem on a variety of *petitio principii* arisen from a *ceteris paribus*[2] clause available to usual inference. That is, the problem is that on the validity of a *ceteris paribus* clause to confirmation of hypothesis from the point of view of epistemology, not from that of mathematics.

2 PROBABILITY AND INDUCTIVE ARGUMENT

We should not make initial probability, an important condition in epistemological inference, fixable to background through time like mathematical case of deductive calculation, especially in inductive argument for confirmation of hypothesis. We first present two definitions in order to solve the problem on the validity of a *ceteris paribus* clause to confirmation of hypothesis from the point of view of epistemology, not from that of mathematics.

DEFINITION 1 (inductive probability as conditional probability-relation). We define $p_{I,n}$ as such inductive probability[3] that means the degree of confirmation of hypothesis H* made up of a premise A and a conclusion H. That is, given evidences $(\prod_{j=0}^{m} E_j)$ in inductive argument for confirmation of hypothesis H*, $p_{I,n}$ stands for the following conditional probability-relation[4] between H and $(A\prod_{j=0}^{m} E_j)$, subject to the logical relation C_n $(n \geqq 0)$ between argument-condition except evidences, and observer.

$$p_{I,n} = P_I(H/(A\prod_{j=0}^{m} E_j)|C_n),\ j=0,1,\ldots \qquad (1)$$

where $\prod_{j=0}^{m} E_j = E_0 E_1 \cdots E_m$.

DEFINITION 2 (argument-condition probability). Next, we define argument-condition probability $p_{a,n}$ as the following conditional probability.

$$p_{a,n}=P_l(C_{n+1}|C_n), \tag{2}$$

where C_0 stands for the logical relation between initial argument-condition except evidences, and observer.

Therefore, the relation between $p_{l,n}$ and $p_{a,n}$ is as follows by (1) & (2). That is,

$$\begin{aligned} p_{l,n} &= P_l(H/(A\prod_{j=0}^{m}E_j)|C_n) \\ &= p_{a,n}P_l(C_n|H/(A\prod_{j=0}^{m}E_j))P_l(H/(A\prod_{j=0}^{m}E_j))/P_l(C_nC_{n+1}). \end{aligned} \tag{3}$$

3 *CETERIS PARIBUS* CLAUSE

Logical form of a *ceteris paribus* clause in inductive argument for confirmation of hypothesis is specified as follows.

COROLLARY 1. A *ceteris paribus* clause first means that the logical relation $(H/A\prod_{j=0}^{m}E_j)$ between H and $(A\prod_{j=0}^{m}E_j)$ is independent of C_n, that is,

$$P_l(H/(A\prod_{j=0}^{m}E_j)|C_n)=P_l(H/(A\prod_{j=0}^{m}E_j)). \tag{4}$$

Let $p^*_{l,n}$ be the $p_{l,n}$ based on the acceptance of a *ceteris paribus* clause in inductive argument for confirmation of hypothesis H^*. Then, by Definition 1,

$$p^*_{l,n}=P_l(H/(A\prod_{j=0}^{m}E_j)). \tag{5}$$

COROLLARY 2. A *ceteris paribus* clause secondly means that $C_0=C_1=\cdots=C_n=C_{n+1}$ in inductive argument for confirmation of hypothesis H^*. Let $p^*_{a,n}$ be the $p_{a,n}$ based on the acceptance of a *ceteris paribus* clause in inductive argument for confirmation of hypothesis H^*. Then, by Definition 2,

$$p^*_{a,n} = P_i(C_0|C_0)$$
$$= 1. \tag{6}$$

And, on the other hand,

$$0 \leqq p_{a,n} = P_i(C_{n+1}|C_n) \leqq 1. \tag{7}$$

Hence, by Corollary 1 & 2,

$$p_{a,n} \leqq p^*_{a,n}. \tag{8}$$

4 *CETERIS PARIBUS* CLAUSE AND INDUCTIVE ARGUMENT

In order to analyze logically the validity of a *ceteris paribus* clause to inductive argument for confirmation of hypothesis, we pay our attention to both difference between $p^*_{i,n}$ and $p_{i,n}$, and difference between $p^*_{a,n}$ and $p_{a,n}$.

First, we use α_n as difference between $p^*_{i,n}$ and $p_{i,n}$.

$$\alpha_n = p^*_{i,n} - p_{i,n}$$
$$= P_i(H/(A\prod_{j=0}^{m}E_j)) - P_i(H/(A\prod_{j=0}^{m}E_j)|C_n)$$
$$= P_i(H/(A\prod_{j=0}^{m}E_j))[1 - P_i(C_n|H/(A\prod_{j=0}^{m}E_j))/P_i(C_n)] \tag{9}$$
$$= P_i(H/(A\prod_{j=0}^{m}E_j))[1 - p_{a,n}P_i(C_n|H/(A\prod_{j=0}^{m}E_j))/P_i(C_nC_{n+1})] \tag{10}$$

(by (3))

It is not always true by (9) that $\alpha_n > 0$. In other words, the acceptance of a *ceteris paribus* clause is not always valid for inductive argument to confirm hypothesis. Whether $\alpha_n > 0$ or not depends on the existence of favorable reality of C_n, i.e., the existence of favorable logical relation C_n between argument-condition except evidences, and observer.

If and only if we have a certain relatively unfavorable logical relation C_n in comparison with acceptance of a *ceteris paribus* clause, we should accept a *ceteris paribus* clause from the point of view of raising the degree of confirmation of hypothesis. In other words, if we have any relatively favorable logical relation C_n, given evidences $(\prod_{j=0}^{m} E_j)$ in inductive argument for confirmation of hypothesis, we had better reject a *ceteris paribus* clause from the point of view of α_n.

Hence, accepting a *ceteris paribus* clause in spite of the existence of some relatively favorable logical relation C_n leads us into an error in inductive argument for confirmation of hypothesis.

Secondly, we define β_n as the difference between $p^*_{a,n}$ and $p_{a,n}$.

$$\beta_n = p^*_{a,n} - p_{a,n}. \qquad (11)$$

By (8),

$$\beta_n \geqq 0. \qquad (12)$$

The acceptance of a *ceteris paribus* clause always contributes inductive argument for confirmation of hypothesis by (12), only from Corollary 2 based on argument-condition probability.

5 TRAP OF A *CETERIS PURIBUS* CLAUSE

In inductive argument for confirmation of hypothesis, if we accept a *ceteris paribus* clause on the condition of the existence of some relatively favorable logical relation C_n, then we will suffer a serious trap arisen from using easily a *ceteris paribus* clause without any epistemological analysis. Because, in this case, unfortunate simultaneity made up of $\alpha_n < 0$ and $\beta_n > 0$ occurs logically by (9) and (12). So, falling into the trap makes a kind of poverty in usual inductive argument for confirmation of hypothesis.

The lack of reality in inductive sciences mentioned in the beginning has been caused eventually both by cutting down original form of conception of inductive probability and by overestimating mathematical transformation from original form of inductive probability, i.e., mathematical probability based on measure axioms. Hence, I think that it is necessary to return to the fundamental insight into the logical and philosophical character of original conception of inductive probability, from the point of view of epistemology of induction, for the purpose of overcoming the lack of reality in inductive sciences.

NOTES

1. Keynes[1921], a young philosopher in Cambridge, was a principal founder of the Cambridge school of probability followed by British empiricism in the first half of the twentieth century, which was derived from the Cambridge New Realism founded by G.E.Moore. (See Harada[1995].) Keynes[1921] conceived probability to be a degree of rational belief in a kind of logical relation extended from normal notion of logical relation made up of a premise A and a conclusion H, i.e., probability-relation $P(H/A)$. Keynes's probability aimed to present the foundations for justification of inductive argument for confirmation of hypothesis H*. Hence Keynes's probability, an original form of conception of probability, had both logical meaning and philosophical meaning, though it was not always measurable

as a mathematical transformation from original form of probability. After Ramsey[1926], another principal representative of the Cambridge school of probability, criticized the ambiguity of Keynes's probability caused both by his conception of probability-relation and by degree of rational belief, Kolmogorov[1933] sophisticated the original conception of probability as mathematically transformed probability based on measure axioms, by cutting down the logical and philosophical character of the original conception of probability. So, the form of mathematical probability, e.g., frequency theory and axiomatist theory, is mathematically transformed from a certain epistemology of probability. Logical interpretation of probability and subjective interpretation of probability are, I think, included in epistemological probability, though the former like Keynes's probability can not always be axiomatized and the latter like Ramsey's probability can be done.

2. Lakatos[1978] says 'one can easily argue that *ceteris paribus* clauses are not exceptions, but the rule in science' (p.18). However, I entertain a serious doubt about it, especially in the case of inductive argument for confirmation of hypothesis. Because, in scientific thinking, *ceteris paribus* clauses are usually used as a kind of simple premise without epistemological consideration for the logical validity.

3. As a proof of impossibility of inductive probability, demonstrating that $P_1(H^*|E_1 b) > P_1(H^*|b)$, an inequality made up of a priori probability and a posteriori probability in the simple case which means that E_1 is deducible from H^* in the presence of background knowledge b, is *an*

illusion, Popper & Miller [1983] proved the justifiability of Hume's basic philosophical problem on induction, which means that the plan for verifying validity of inductive process falls into *petitio principii*. However, what is background knowledge b? What we should solve on Hume's problem hereafter, I think, is to verify validity of the inequality again, by the extension of background knowledge b to the logical relation C_n above-mentioned between argument-condition except evidences, and obserber. Concerning the application of new fundamental theory of confirmation of hypothesis, as a view of inductive logic related to inductive probability, like Putnam[1975] says that 'the task of inductive logic is the construction of a *universal learning machine*' (p.303), I also think that the resurrection of logic of partial belief introduced by Ramsey[1926] should be applied to Artificial Intelligence.

4. Popper[1959] discussed the problem on measurability of confirmation function $C(H^*|E_1b)$, and he revealed that $C(H^*|E_1b) \neq P(H^*|E_1b)$ given probability function $P(\cdot)$. (See Gillies[1995] & Simkin[1993].) I think that we must closely examine validity on the equivalent relation between confirmation function and probability function like an argument by Gillies[1991], who regards theory of confirmation as 'an application to a central area of philosophy of science'(p.518). However, we here *provisionally* assume justifiability of the following equality based on probability-relation, which is derived from Bayesian thesis:

$$P_1(H/(A\prod_{j=0}^{m}E_j)|C_n) = C_1(H/(A\prod_{j=0}^{m}E_j)|C_n).$$

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