

経済と経営 21-3 (1991. 1)

〈研究ノート〉

A Note on the Condition for Optimum Tariff and Subsidy*

Masayuki Hayashibara

In a simplified case, criteria for optimum tariff (subsidy) by Brander and Spencer and by Jones are equivalent to each other and the Metzler paradox cannot occur. In a more general case with general equilibrium income effects of tariff, a necessary condition for the Metzler paradox is that importables must be strongly inferior and/or the tariff can reduce extremely the slope of the demand curve. So we may safely rule it out under the normal goods assumption.

1. Introduction

Brander and Spencer (1984) showed that one country facing a foreign monopolistic supplier of its imports could subsidize rather than tax importables in order to maximize its benefit in the ad valorem regime and

* I would like to thank Ronald W. Jones and Shumpei Takemori for their helpful suggestions.

the specific one¹⁾. We will reconsider their condition under which optimum rate of tariff could be negative (a subsidy), in relation to the partial effect of tariff on a price elasticity of demand at the initial price.

From section 2 to 4 we limit ourself to a simplified case, that is, a partial equilibrium analysis with the demand function $D(p)$ where p denotes domestic price of importables, no domestic production and segmented domestic market. A more general case of demand function such as $D(p, T)$ where general equilibrium income effects of tariff T are incorporated will be considered in section 5.

Now there exist the following criteria for optimum tariff, no intervention or subsidy, (i) by the elasticity or relative curvature of demand function²⁾ and (ii) by the partial effect of tariff change on a price elasticity of demand at the initial price³⁾.

In this note, these two criteria will be shown equivalent to each other in a simplified case. Further the Metzler paradox where domestic price of importables declines due to tariff cannot occur in the simplified case. In a more general case with general equilibrium income effects of tariff, a necessary condition for the Metzler paradox is that importables must be strongly inferior goods and/or tariff can reduce extremely the slope of demand curve at the initial domestic price. Therefore, while it might be possible for the Metzler paradox to emerge, we may safely rule it out under the normal goods assumption.

1) But they wrote "In both regimes the subsidy case requires strong convex demand and would have to be considered unusual" (p. 241).

2) Brander and Spencer (1984).

3) Jones (1987).

2. Preliminaries

Suppose a market for importables in a partial equilibrium analysis. Let the demand function be $D(p)$ where p denotes domestic price and assume no domestic production. Let the price charged by foreign monopolist be p^* and tariff parameters by home government be T . Then a general formula for domestic price determination can be written as :

$$(2-1) \quad p = p(p^*, T),$$

with partial derivatives $\delta p / \delta p^* > 0$ and $\delta p / \delta T > 0$. From the demand function we get a price elasticity of demand for importables as $\epsilon(p) = -pD'(p)/D(p)$ where prime indicates derivative. Next consider the effects of tariff change on the price elasticity. In general,

$$(2-2) \quad \begin{aligned} d\epsilon/dT &= (\delta\epsilon/\delta p^*) dp^*/dT + \delta\epsilon/\delta T \\ &= \epsilon'(\delta p/\delta p^*) dp^*/dT + \epsilon' \delta p/\delta T \end{aligned}$$

must hold. That is, the price elasticity depends on domestic price (in addition to preference and income which are given by assumption) which in turn depends on supply price and tariff parameters. We are especially interested in the latter relation.

3. Ad valorem tariff regime

This section reviews the case of ad valorem tariff so put,

$$(3-1) \quad p = (1+T)p^*.$$

Maximizing behavior of the monopolistic foreign firm can be described as : maximizing profit $\pi^*(p^*) = (p^* - c^*)D(p)$ with respect to p^* for given T where c^* stands for its constant marginal cost. The first order condition

implies that optimum price charged by the foreign firm becomes :

$$(3-1) \quad p^* = c^* \varepsilon / (\varepsilon - 1).$$

And the second order condition will be :

$$(3-2) \quad 2D' + (p^* - c^*)(1+T)D'' < 0,$$

which in turn implies, assuming $1+T > 0$:

$$(3-3) \quad A = (1+T)c^*\varepsilon' + (\varepsilon - 1)^2 > 0.$$

Next consider $d\varepsilon/dT$. In the ad valorem tariff case it can be reduced to $(1+T)\varepsilon' dp^*/dT + \varepsilon p^*$. But this expression is nothing but the total effect of tariff changes on the price elasticity of demand and it is composed of two parts, in particular it includes dp^*/dT which should be determined endogenously. Totally differentiating (3-2) yields :

$$(3-4) \quad dp^*/dT = -(c^* d\varepsilon/dT) / (\varepsilon - 1)^2$$

from which we can, using decomposition, get :

$$(3-5) \quad dp^*/dT = -c^* p^* \varepsilon' / A.$$

Now consider the Brander-Spencer condition, that is :

$$(3-6) \quad \begin{aligned} dp^*/dT > 0 & \quad \text{if } \eta_X > 0 \text{ (subsidy),} \\ dp^*/dT = 0 & \quad \text{if } \eta_X = 0 \text{ (no intervention),} \\ dp^*/dT < 0 & \quad \text{if } \eta_X < 0 \text{ (tariff),} \end{aligned}$$

where the inverse demand function is $p = p(X)$, $\eta = -p/[Xp'(X)]$ and $\eta_X = d\eta/dX$. Simple calculation shows that this condition is equivalent to :

$$(3-7) \quad \begin{aligned} dp^*/dT > 0 & \quad \text{if } p^* \varepsilon' < 0 \text{ (subsidy),} \\ dp^*/dT = 0 & \quad \text{if } p^* \varepsilon' = 0 \text{ (no intervention),} \\ dp^*/dT < 0 & \quad \text{if } p^* \varepsilon' > 0 \text{ (tariff).} \end{aligned}$$

Thus because the effect of tariff on the price elasticity of demand at the initial price can be $p^* \varepsilon'$ in ad valorem regime, the condition whether or not the tariff improves the country's terms of trade depends upon the elasticity along the demand curve behavior and this in turn can be

expressed as the effect of tariff on the elasticity of demand at the initial international price as shown by Jones⁴.

Next, to analyze the condition for the Metzler paradox where a tariff reduces domestic price of importables, we put :

$$(3-9) \quad (dp^*/dT)/p^* < -1/(1+T) \text{ and } T \geq 0,$$

which is equivalent to an inequality $0 > (\epsilon - 1)^2$. Because this cannot hold, however, the Metzler paradox can be ruled out.

4. Specific tariff regime

Next, consider the case of specific tariff where domestic price are represented by,

$$(4-1) \quad p = p^* + T.$$

By the first order condition for profit maximization optimum price set by the foreign firm is :

$$(4-2) \quad p^* = (T + c^* \epsilon) / (\epsilon - 1),$$

where as in section 3, c^* stands for its constant marginal cost. And the second order condition must be :

$$(4-3) \quad 2D' + (p^* - c^*)D'' < 0,$$

which in turn implies, by assuming $T + c^* > 0$:

$$(4-4) \quad B = (\epsilon - 1)^2 + (T + c^*) \epsilon' > 0.$$

Because $d\epsilon/dT$ can be reduced to $\epsilon' dp^*/dT + \epsilon'$ in the specific tariff regime, the effect of tariff on the price elasticity at the initial price is ϵ' .

4) Proposition by Jones is that if the demand schedule facing a monopolist shifts in (down) such that the elasticity of demand at the old equilibrium price has been reduced, the new equilibrium price is higher than the old. Jones (1987) p. 377.

Totally differentiating (4-2) yields :

$$(4-5) \quad dp^*/dT = \{(\epsilon - 1) - (T + c^*)d\epsilon/dT\} / (\epsilon - 1)^2,$$

using decomposition we get :

$$(4-6) \quad dp^*/dT = \{(\epsilon - 1) - (T + c^*)\epsilon'\} / B.$$

By the second order condition the denominator must be positive.

Now consider the Brander-Spencer condition :

$$(4-7) \quad dp^*/dT < 0 \quad \text{if } R > -1 \text{ (tariff),}$$

$$dp^*/dT = 0 \quad \text{if } R = -1 \text{ (no intervention),}$$

$$dp^*/dT > 0 \quad \text{if } R < -1 \text{ (subsidy),}$$

where we let the inverse demand function be $p = p(D)$ and a measure of the relative curvature of the demand curve be $R = p''D/p'$. Again simple calculation shows that this condition is equivalent to :

$$(4-8) \quad dp^*/dT < 0 \quad \text{if } \epsilon' > (\epsilon - 1)/(T + c^*) \text{ (tariff),}$$

$$dp^*/dT = 0 \quad \text{if } \epsilon' = (\epsilon - 1)/(T + c^*) \text{ (no intervention),}$$

$$dp^*/dT > 0 \quad \text{if } \epsilon' < (\epsilon - 1)/(T + c^*) \text{ (subsidy).}$$

Thus, the above argument shows that condition whether or not the specific tariff improves the country's terms of trade depends upon the curvature of the demand curve and this in turn expressed as the effect of tariff on the elasticity of demand at the initial price. (4-8) indicates that the constant price elasticity which is greater than one implies optimum subsidy⁵⁾.

Finally to analyze the Metzler paradox put,

$$(4-9) \quad dp^*/dT < -1 \text{ and } T > 0.$$

However inequality (4-9) is reduced to $\epsilon(\epsilon - 1) < 0$ which implies $0 < \epsilon < 1$

5) Brander and Spencer (1984) p. 232. However, the constant elasticity in ad valorem regime implies no intervention as shown by Brander and Spencer.

that contradicts a positive price. Thus the Metzler paradox cannot occur in this case either.

5. Generalization

Here we assume a more general demand function with general equilibrium income effects of tariff and consider the condition under which the Metzler paradox might occur. We analyze only a specific regime. Let a reduced form demand function for importables be :

$$(5-1) \quad D(p, T) \quad \text{where} \quad p = p^* + T,$$

and the price elasticity of demand can be written as :

$$(5-2) \quad \epsilon(p, T) = -pD_p/D,$$

where subscript stands for partial derivative. Thus change in tariff brings about an additional effect on the elasticity.

$$(5-3) \quad d\epsilon/dT = \epsilon_p dp^*/dT + \epsilon_p + \epsilon_T.$$

The foreign firm maximizes its profit :

$$(5-4) \quad \pi^*(p^*) = (p^* - c^*)D(p, T).$$

Then from the first order condition, optimum price set by the foreign firm can be given as :

$$(5-5) \quad p^* = (T + c^*\epsilon)/(\epsilon - 1).$$

And the second order condition must be, by assuming $T + c^* > 0$:

$$(5-6) \quad C = (\epsilon - 1)^2 + (T + c^*)\epsilon_p > 0.$$

Because totally differentiating (5-5) yields :

$$(5-7) \quad dp^*/dT = \{(\epsilon - 1) - (T + c^*)(\epsilon_p + \epsilon_T)\}/C,$$

criterion becomes :

$$(5-8) \quad \begin{aligned} dp^*/dT < 0 & \quad \text{if} \quad \epsilon_p + \epsilon_T > (\epsilon - 1)/(T + c^*), \\ dp^*/dT > 0 & \quad \text{if} \quad \epsilon_p + \epsilon_T < (\epsilon - 1)/(T + c^*). \end{aligned}$$

Condition whether or not the specific tariff improves the country's terms of trade depends upon the sum of the effect of tariff on the elasticity of demand at the initial international price, ϵ_p , and general equilibrium income effects of tariff on it, ϵ_T . Again the price elasticity of demand that is constant but greater than one implies optimum subsidy.

Finally, to consider the Metzler paradox, put $dp^*/dT < -1$ and $T > 0$, which can be reduced to :

$$(5-9) \quad \epsilon_T > (\epsilon - 1) \epsilon / (T + c^*).$$

To assure a positive price we assume $\epsilon > 1$, then the right-hand-side is positive which implies :

$$(5-10) \quad \epsilon_T = (D_p D_T - D D_{pT}) p / D^2 > 0.$$

Thus a necessary condition for the Metzler paradox is that importables must be strongly inferior goods and/or, the tariff can reduce extremely the slope of the demand curve at the initial domestic price. D_T is equal to $mD/(p-mT)$ where m denotes the marginal propensity to consume importables⁶. In fact, under $T=0$ initially, (5-9) can be equivalent to :

$$(5-11) \quad m - (p D_{pT} / D_p) < -\epsilon < -1.$$

6) This can be gotten from an ordinary demand function :

$$(1) \quad D = D(p, y),$$

where y stands for real income whose changes will be :

$$(2) \quad dy = -D dp^* + T dD.$$

Totally differentiating (1) yields :

$$(3) \quad p dD = -D \epsilon dp + m dy,$$

where ϵ and m indicate the compensated elasticity of importables and the marginal propensity to consume it, respectively. Substituting (2) into (3) and using $dp^* = dp - dT$, we have :

$$(4) \quad (p - mT) dD = -D(\epsilon + m) dp + m D dT.$$

Thus, while it might be possible for the Metzler paradox to emerge, we can rule it out safely under the normal goods assumption.

6. Concluding remarks

In a simplified case, two criteria for optimum tariff, no intervention and optimum subsidy, that is, the elasticity or relative curvature of the demand function by Brander and Spencer, and the partial effect of tariff change on the price elasticity of demand at the initial price by Jones have been shown to be equivalent to each other. Further, the Metzler paradox where domestic price of importables declines due to tariff cannot occur in the simplified case. In a more general case such as Jones and Takemori (1989), which contains among other things general equilibrium income effects of tariff, a necessary condition for the Metzler paradox is that importables must be strongly inferior and/or tariff can reduce extremely the slope of demand curve at the initial price. So we may safely rule out the Metzler paradox under the normal goods assumption.

References:

- Brander J. A. and B. J. Spencer (1984), "Trade Welfare : Tariffs and Cartels", *Journal of International Economics*, Vol. 16, May, pp. 227–242.
- Jones R. W. (1987), "Trade Taxes and Subsidies with Imperfect Competition", *Economics Letters*, Vol. 23, pp. 375–379.
- Jones R. W. and S. Takemori (1989), "Foreign Monopoly and Optimal Tariffs for the Small Open Economy", *European Economic Review*, Vol. 33, December, pp. 1691–1708.